

# Comment on the Public Key Substitution Attacks\*

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## Abstract

In this paper we present a comment on some previous works about the Public Key Substitution Attacks (PKSA in brief). Though there exist some security flaws for the schemes being attacked, we point out that these attacks on them are either trivial or avoidable after a little modification.

*Keywords:* Public key cryptography, public key substitution attacks, public key authentication, proxy signature, key agreement

## 1 Introduction

The concept of Public Key Cryptography, introduced by Diffie and Hellman in 1976 [5], may be a milestone in the history of cryptology. In public key systems, each user has two different keys: a public key and a private key. Therefore, the capacities for encryption and decryption are separated. This separation allows important improvement in the key management and makes it possible for digital signature [4].

In public key systems, all public keys are usually stored in a public database called a public key directory. The public key directory is very important yet vulnerable. An intruder can impersonate a target user by substituting his public key with a forged public key, called Public Key Substitution Attacks (PKSA in brief). Therefore, the public key should be verified to prevent this attack before communicating. Depending on the ways of verifying the public key, the public key systems can be categorized as follows: Certificate-based public key systems [8], Identity-based public key systems [15] and Self-certified public key systems [7].

Though PKSA are dangerous events for public key systems, we found some previous works about PKSA, including attacks on a public key authentication scheme, some

proxy signature schemes and Station-to-Station Message Authentication Code (STS-MAC) protocol, are either trivial or avoidable after a little modification.

The rest of the paper is organized as follows: Some preliminaries are given in Section 2. The comment on PKSA on a public key authentication scheme is given in Section 3. The comment on PKSA on some proxy signature schemes is given in Section 4. The comment on PKSA on STS-MAC protocol is given in Section 5. Finally, conclusions will be made in Section 6.

## 2 Preliminaries

In this section, we will briefly introduce the public key substitution attack in certificate-based public key systems.

In certificate-based public key systems, a highly trusted organization, i.e., Certificate Authority (CA) issues a witness  $\mathcal{W}$ , often called certificate, which binds the user's identity  $\mathcal{I}$  to the public key  $\mathcal{P}$ . The certificate format usually adopts CCITT X.509 standard [3]. Everyone can be convinced that  $\mathcal{P}$  is really the public key of  $\mathcal{I}$  by the CA's digital signature. The disadvantages of the systems is the requirement of the additional cost of computation and storage for  $\mathcal{W}$  and  $\mathcal{P}$ .

There exists a security flaw called Public Key Substitution Attacks (in brief, PKSA) for Certificate-based systems. If a dishonest CA wants to impersonate the user whose public key is  $pk$ . He can do as follows:

- He randomly chooses a different secret key  $sk'$  and computes the corresponding public key  $pk'$ .
- He then generates a certificate  $C'$  which binds the users' identity information  $ID$  with  $pk'$ . Obviously, CA can impersonate the user to communicate with others.

However, the user can accuse the dishonest CA because there exist his two different "valid" certificates issued by

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the CA for a same period  $T$ . The arbiter deduces that CA is dishonest because no one could forge CA's signature.

Therefore, Certificate-based systems reach Girault's trusted level 3, *i.e.*, the authority does not know the private key of the users, and it can be proven that the authority generates false witness if he does so. Consequently, we usually omit this security flaw when using Certificate-based systems.

### 3 PKSA on a Public Key Authentication Scheme

In 2003, Lee, Hwang and Li [10] proposed a new public key authentication scheme for cryptosystems with a trusty server. Their scheme is based on discrete logarithm too, and in their scheme, the certificate of the public key is a combination of user's password and private key. Later, Zhang and Kim [18] showed that Lee-Hwang-Li's key authentication scheme is not secure. From the obtained public information, any one can get the private key of the user. Furthermore, they proposed an improved scheme. Recently, Sun, Cao and Sun [14] showed that Zhang-Kim's proved scheme still suffered from public key substitution attack, *i.e.*, a dishonest user can forge his public key via the verification equation. Therefore, they claimed that Zhang-Kim's scheme cannot achieve the non-repudiation service like the previous variants.

In this section, we point out that their attack is trivial, which is same to PKSA in Certificate-based systems. First of all, we overview Zhang and Kim's scheme and Sun-Cao-Sun's PKSA on it.

#### 3.1 Zhang and Kim's Scheme

The system parameters of the scheme are as follows: Let  $p$  and  $q$  be prime numbers such that  $q|p-1$ ,  $g$  is a generator with order  $q$  in  $Z_p^*$ . The one-way function  $f$  is defined by  $f(x) = g^x \bmod p$ . The user of the system has  $Prv$  as his/her private key and  $PWD$  as his/her password. Let  $Pub = g^{Prv} \bmod p$  be the user's public key.

- Registration:

In the registration phase, each user chooses a random number  $r \in Z_q^*$ , and calculates  $f(PWD + r)$ . The user then himself generates his public key certificate  $C = PWD + r + Prv \times Pub \bmod q$ .

The user then sends  $f(PWD + r)$ ,  $R = g^r \bmod p$  and his ID to the server secretly. The server then verifies if  $f(PWD + r) = f(PWD) \times R$  and verifies the  $f(PWD + r)$  sent by the legal user, and then stores ID and  $f(PWD + r)$  in public password table in the server. The public password table cannot be modified or forged by an attacker because the server can use the technique of access control to protect it. The certificate  $C$  and public key  $Pub$  of the user are opened to the public over the network.

- Authentication:

In the key authentication phase, when someone wants to communicate with a user, the sender first obtains  $C$ ,  $Pub$  and  $f(PWD + r)$  of the receiver from the public directory in the network and public password table in the server, and then checks the certificate  $C$  of the public key of the receiver by computing the following equation:

$$f(C) = f(PWD + r) \times Pub^{Pub} \bmod p.$$

If and only if the above equation holds, the sender accepts the public-key  $Pub$  of the receiver.

#### 3.2 Sun-Cao-Sun's PKSA

In Zhang and Kim's scheme, assume that a dishonest legal user who has a key pair  $(prv, pub)$  uses his private key  $prv$  to generate his signature for a document. Now anyone can verify this signature using his public key  $pub$ . To deny his signature, the dishonest user does as follows:

- 1) Arbitrarily selects a forged private key  $prv'$  such that  $prv' \neq prv \bmod q$ , and then computes the corresponding forged public key  $pub' = g^{prv'} \bmod p$ .
- 2) Computes the forged certificate  $C'$  as follows:

$$C' = pwd + r + prv' \times pub' \bmod q.$$

- 3) Substitutes the fabrication  $C'$  and  $pub'$  in the public key directory.

Since the dishonest user knows his own password  $pwd$  and random number  $r$ , he can easily obtain the forged certificate  $C'$ . Now anyone will also be convinced of the integrity of the forged public key  $pub'$ .

#### 3.3 Comment

Sun, Cao and Sun claimed that the signature generated using  $prv$  cannot be verified using the forged public key  $pub'$ . As a result, the dishonest user can deny his signature. However, when a dispute occurs, the receiver can send the signature together with the public key  $pub$  and corresponding certificate  $C$  to the arbiter to accuse the dishonest user. If the certificate  $C$  ensures the validity of the public key  $pub$  and the signature can be verified by this public key  $pub$ , the arbiter can deduce the user dishonest because only the user can generate a valid certificate for himself. Actually, note that the equation  $g^C/pub^{pub} = g^{C'}/pub'^{pub'}$  holds, the arbiter can easily deduce the dishonest user generated the forged certificate and corresponding public key.

This is the same as Certificate-based systems. For example, a user generated his key pair  $(sk, pk)$  and signed messages with the private key  $sk$ , where the validity of  $pk$  is ensured by a certificate  $C$  issued by a trusted CA. After a period of time  $T$ , he updated his key pair  $(sk', pk')$  and got a new certificate  $C'$  from the CA. Now he uses the

new secret key  $sk'$  to generate signatures which can be verified by  $pk'$ . However, he still could not deny his any signature signed by the private key  $sk$ .

## 4 PKSA on Some Proxy Signature Schemes

In 2001, Lee, Kim and Kim proposed a non-designated proxy signature scheme [11] and used it to design secure mobile agents in electronic commerce [12]. Many researchers presented the security analysis of this scheme [16, 9, 17]. Since Wang et al's analysis is a general case and the others can be regarded as a special case of it, we only focus on it in this paper.

### 4.1 Lee-Kim-Kim's Scheme

The system parameters of the scheme are as follows: Let  $p$  and  $q$  be prime numbers such that  $q|p-1$ ,  $g$  is a generator with order  $q$  in  $Z_p^*$ . Let  $h$  be a cryptographic secure hash function. Suppose Alice be the original signer and Bob be the proxy signer. Each entity  $I$  has a certified key pair  $(x_I, y_I)$ , here  $x_I$  is the private key and  $y_I = g^{x_I}$  is the corresponding public key.

- Proxy Key Generation:

The original signer Alice uses the Schnorr scheme [13] to sign a warrant  $m_w$ , which specifies which messages Bob can sign on behalf of Alice, the validity period of delegation, etc. That is, Alice chooses a random number  $k_A \in Z_q$ , computes  $r_A = g^{k_A} \bmod p$  and  $s_A = k_A + x_A + x_A h(m_w, r_A) \bmod q$ . Then, the tuple  $(m_w, r_A, s_A)$  is sent to the proxy signer Bob, and Bob checks its validity by  $g^{s_A} = y_A^{h(m_w, r_A)} \cdot r_A \bmod p$ .

If this verification is correct, Bob sets his proxy key pair  $(x_P, y_P)$  as  $x_P = s_A + x_B \bmod q$ ,  $y_P = g^{x_P} \bmod p = y_A^{h(m_w, r_A)} \cdot r_A \cdot y_B \bmod p$ .

- Proxy Signature Generation:

With the proxy key pair  $(x_P, y_P)$ , Bob can use any DLP-based signature scheme to generate proxy signature on any delegated message  $m$ . The resulting proxy signature is a tuple  $\sigma = (\text{sign}(m, x_P), m_w, r_A, y_A, y_B)$ .

- Proxy Signature Verification:

To check the validity of  $\sigma$ , a verifier first checks whether the message  $m$  conforms to the warrant  $m_w$ . He then computes the proxy public key  $y_P = y_A^{h(m_w, r_A)} \cdot r_A \cdot y_B \bmod p$ . Finally, the verifier checks whether  $\text{sign}(m, x_P)$  is a valid signature on message  $m$  with respect to the proxy public key  $y_P$  in the corresponding DLP-based signature scheme.

### 4.2 Wang-Bao-Zhou-Deng's PKSA

The original signer Alice can mount the following attack:

- Create a warrant  $\bar{m}_w$ , select a random number  $c \in Z_q^*$ , and then define  $\bar{r}_A = y_B^{-1} g^c \bmod p$ .

- The forged proxy key pair  $(\bar{x}_P, \bar{y}_P)$  is given by:

$$\bar{x}_P = c + x_A h(\bar{m}_w, \bar{r}_A) \bmod q, \quad \bar{y}_P = g^{\bar{x}_P} \bmod p.$$

- Anyone can directly verify that  $(\bar{x}_P, \bar{y}_P)$  is a valid proxy key pair with respect to  $(\bar{m}_w, \bar{r}_A)$ , since  $\bar{y}_P = g^{\bar{x}_P} \bmod p = y_A^{h(\bar{m}_w, \bar{r}_A)} \cdot \bar{r}_A \cdot y_B \bmod p$ .

### 4.3 Comment

In certificate-based systems, the validity of the users' public key  $pk$  is ensured by the signature  $\sigma$  of CA. In Lee, Kim and Kim's proxy signature scheme, note that the proxy public key  $y_P = y_A^{h(m_w, r_A)} \cdot r_A \cdot y_B \bmod p = g^{s_A} \cdot y_B \bmod p$ , i.e., the validity of  $y_P$  is ensured by the signature  $(r_A, s_A)$  of the original signer.

When a dispute between the original signer and the proxy signer occurs, the proxy signer presents the object proxy signature  $\sigma = (\text{sign}(m, x_P), m_w, r_A, y_A, y_B)$  to the arbiter. The arbiter first computes the corresponding proxy public key  $y_P$ , and then checks whether  $\text{sign}(m, x_P)$  is a valid signature on message  $m$  with respect to  $y_P$ . If the check is negative, he claims that the original signer is dishonest. Otherwise, he asks the original signer to provide  $s_A$  which satisfies  $g^{s_A} = y_A^{h(m_w, r_A)} \cdot r_A \bmod p$ . If the original signer can not provide such a value, he deduces that the original signer is dishonest. Otherwise, he deduces that the proxy signer generated the signature. This requires the original signer to keep the value  $s_A$ , just as CA should keep the users' certificate in Certificate-based systems. We argue that it should be an implicit condition though this is never mentioned in the Lee-Kim-Kim's scheme. On the other hand, it is not desirable in situations where the storage is small.

Furthermore, wang et al. showed that the revised version of Lee-Kim-Kim's scheme [9] also suffered from the PKSA by the dishonest original signer. Our above comment is also suitable for this attack.

## 5 PKSA on Key Agreement Protocol

It is well-known that PKSA can also be used in key agreement protocol. First, we introduce this attack on Station-to-Station Message Authentication Code (STS-MAC) protocol.

### 5.1 PKSA on STS-MAC Protocol

Let  $A$  is the initiator and  $B$  responder. In the attack against the responder, the adversary  $E$  registers  $A$ 's public key  $P_A$  as its own; i.e.,  $P_E = P_A$ . When  $A$  sends  $B$  message (1),  $E$  intercepts it and replaces the identity  $I_A$  with  $I_E$ .  $E$  then passes message (2) from  $B$  to  $A$

unchanged. Finally  $E$  intercepts message (3), and replaces  $Cert_A$  with  $Cert_E$ . Since  $P_A = P_E$ , we have  $S_A(\alpha^{r_A}; \alpha^{r_B}) = S_E(\alpha^{r_A}; \alpha^{r_B})$ . Hence  $B$  accepts the key  $K$  and believes that  $K$  is shared with  $E$ , while in fact it is shared with  $A$ . Note that  $E$  does not learn the value of  $K$ . The attack is depicted below. The notation  $A \leftrightarrow B$  means that  $A$  transmitted a message intended for  $B$ , which was intercepted by the adversary  $E$  and not delivered to  $B$ .

- 1)  $A \leftrightarrow B : I_A, \alpha^{r_A}$
- 1')  $E \leftrightarrow B : I_E, \alpha^{r_A}$
- 2)  $E \leftrightarrow B : Cert_B, \alpha^{r_B}, S_B(\alpha^{r_B}, \alpha^{r_A}), MAC_K(S_B(\alpha^{r_B}, \alpha^{r_A}))$
- 2')  $A \leftrightarrow E : Cert_B, \alpha^{r_B}, S_B(\alpha^{r_B}, \alpha^{r_A}), MAC_K(S_B(\alpha^{r_B}, \alpha^{r_A}))$
- 3)  $A \leftrightarrow B : Cert_A, S_A(\alpha^{r_A}, \alpha^{r_B}), MAC_K(S_A(\alpha^{r_A}, \alpha^{r_B}))$
- 3')  $E \leftrightarrow B : Cert_E, S_A(\alpha^{r_A}, \alpha^{r_B}), MAC_K(S_A(\alpha^{r_A}, \alpha^{r_B}))$

Similarly,  $E$  can perform an attack against the initiator  $A$ . For details, see [1].

## 5.2 Comment

The reason why PKSA occur in STS-MAC protocol is that CA issued two “valid” certificates which correspond to a same public key. The possibility for two different users to choose the same private key is negligible. Consequently, the public key for each user should be different.

It is obvious that this attack can be prevented by requiring that each entity proves to CA possession of a private key corresponding to his public key during the certificate issuing process [1]. Actually, in CCITT X.509 standard, a user randomly chooses his secret key  $sk$ , and computes the corresponding public key  $pk$ . He then sends  $pk$  to the authority and proves to the authority that he knows  $sk$  without revealing it. If the proof holds, the authority issued a certificate to the user. Therefore, PKSA in STS-MAC protocol can always be avoidable.

## 6 Conclusions

In this paper we present a comment on some previous works about the Public Key Substitution Attacks, including attacks on a public key authentication scheme, some proxy signature schemes and STS-MAC protocol. Though there exist some security flaws for the schemes being attacked, we point out that these attacks on them are either trivial or avoidable with a little modification.

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