

Notes on “Polynomial-based Key Management for Secure Intra-Group and Inter-Group Communication”

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Abstract

In 2012, Piao et al. proposed a polynomial-based key management for secure intra-group and inter-group communication. In this notes, we point out that there are some security weaknesses of Piao et al.’s intra-group key distribution scheme. One main problem is that their scheme cannot prevent a group member to obtain other members’ secret keys shared with the controller. In addition, their scheme is suffered from the replay attack and cannot achieve the objectives of both perfect forward and backward secrecy. We provide a simple modified scheme to overcome these security weaknesses.

Keywords: Perfect forward and backward secrecy, polynomial-based key distribution, replay attack, secure intra-group communication

1 Introduction

Group communication has been widely developed in various communication applications and environments such as conferences [1, 6], wireless sensor networks [2, 7, 10, 14], and ad hoc networks [5]. In a secure group communication, a dynamic group key needs to be shared among all group members. All group members will use the group key to protect their information. The polynomial-based key distribution [1, 2, 7, 8, 11, 12, 13] is one of the group key distributions. Wang et al. [11, 12, 13] proposed a polynomial-based inter-group key sharing to use a *controller* to distribute personal key shares for inter-group communication. As shown in Figure 1, an l -degree polynomial $P_{ij}(x)$ constructed by the controller in G_i is used to distribute a secret group key such that a node (i.e., the variable x) in G_j and members of G_i can communicate with each other using the key. In 2012, Piao et al. [8] proposed a more efficient generation method of $P_{ij}(x)$ based on [11, 12, 13] such that each member can construct $P_{ij}(x)$ by her/himself. They also developed another polynomial $F(x)$

for intra-group key distribution such that all group members can efficiently retrieve the intra-group key from the broadcast message sent by the controller. Unfortunately, in this notes, we point out that there are some security weaknesses of Piao et al.’s intra-group key distribution scheme [8]. One main problem is that their scheme cannot prevent a group member to obtain other members’ secret keys shared with the controller. Furthermore, their scheme is suffered from the replay attack and cannot achieve the objectives of both perfect forward and backward secrecy.

The remainder of this paper is organized as follows. In Section 2, we briefly review Piao et al.’s intra-group key sharing and re-keying, followed by the cryptanalysis on their scheme in Section 3. Subsequently, we provide a simple modified scheme and its detailed security analysis in Sections 4 and 5, respectively. Finally, we make conclusions in Section 6.

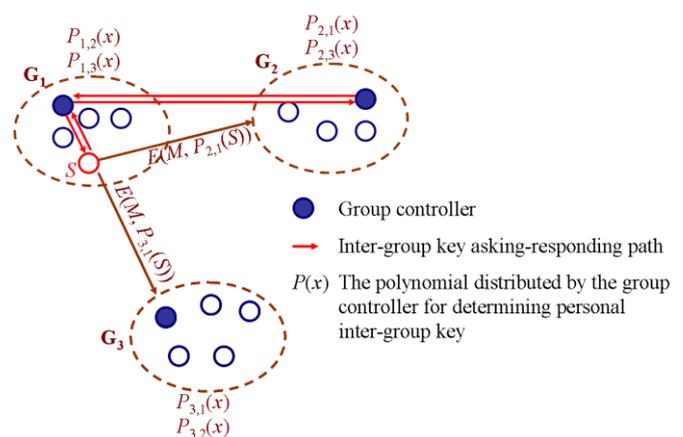


Figure 1: An example of Wang et al.’s inter-group key sharing: a member S in G_1 sends message to all of members in both G_2 and G_3 .

2 Piao et al.'s Scheme

Here, we briefly review the key management scheme of intra-group key sharing and re-keying proposed in [8]. The notations used in this scheme are defined as follows.

G_k : the k -th group

GK_k : the intra-group key for members of G_k

SK_t : the pre-distributed secret key shared between the group controller and a member t in the same group

$F(x)$: the polynomial function in a finite field $GF(p)$ used for deriving intra-group key GK_k , where p is a large prime

2.1 Intra-Group Key Sharing

In a group G_k with n members, assume that each member has pre-shared a secret key SK_t with the group controller through a secure channel. If the group controller wants to distribute the intra-group key GK_k to all members for a secure group communication, the controller has to perform the following procedures.

Step 1. The group controller selects an intra-group key GK_k for all members in the group.

Step 2. The group controller uses all secret keys shared with the group members to generate a polynomial $F(x)$ and conceal the intra-group key GK_k in it i.e., $F(x) = (x-SK_1)(x-SK_2)\dots(x-SK_n)+GK_k$. The controller then broadcasts $F(x)$ to all members.

Step 3. Upon receiving the polynomial $F(x)$, every member can use her/his own secret key SK_t to retrieve the intra-group key GK_k by computing $F(SK_t)$.

2.2 Re-Keying

If there is any change in the group membership, the group controller has to renew the intra-group key for the sake of forward/backward secrecy. Based on the example given in Subsection 2.1, the re-keying for member joining and leaving are described in the following processes, respectively.

2.2.1 For Member Joining

Assume that a new member W wants to join in the group G_k . The group controller has to perform the following procedures.

Step 1. The group controller gives a share key SK_W to W in a secure channel.

Step 2. The group controller generates a new intra-group key GK'_k and constructs a new polynomial as $F'(x) = (x-SK_1)(x-SK_2)\dots(x-SK_n)(x-SK_W)+GK'_k$. The controller then broadcasts $F'(x)$ to all members.

Step 3. Upon receiving the polynomial, every member can use her/his own secret key SK_t to retrieve the intra-

group key GK'_k by computing $F'(SK_t)$.

2.2.2 For Member Leaving

If a member Z leaves the group G_k , the group controller has to perform the following procedures.

Step 1. The group controller generates a new intra-group key GK'_k and constructs a new polynomial as $F'(x) = (x-SK_1)(x-SK_2)\dots(x-SK_{Z-1})(x-SK_{Z+1})\dots(x-SK_n)+GK'_k$. The controller broadcasts $F'(x)$ to members remained in the group.

Step 2. Upon receiving the polynomial, every member can use her/his own secret key SK_t to retrieve the intra-group key GK'_k by computing $F'(SK_t)$.

3 Security Problems of Piao et al.'s Scheme

In this section, we analyze Piao et al.'s scheme [8] and point out some security problems.

3.1 One-Time Use of Pre-Shared Secrets

In [8], the polynomial $F(x)$ in a finite field $GF(p)$, where p is a large prime, is used to distribute the intra-group key. As pointed out in [4], if the modulus used for the polynomial operation is a prime, it may suffer from the so-called *internal attack*. The internal attack is launched by a legitimate group member who knows the group key. But, the attacker tries to obtain the secrets of other group members shared with the controller. For example, if a dishonest member, Ivy, in a group has received a polynomial $F(x) = (x-SK_1)(x-SK_2)\dots(x-SK_t)\dots(x-SK_n)+GK_k$ from the group controller and retrieved the intra-group key GK_k , she can further deduce another polynomial in $GF(p)$ as $H(x) = (F(x)-GK_k)/(x-SK_t) = (x-SK_1)(x-SK_2)\dots(x-SK_{t-1})(x-SK_{t+1})\dots(x-SK_n)$. It is computationally feasible for solving the roots of the polynomial $H(x)$, which are other members' secret keys SK_t 's. Thus, the pre-shared secrets of members in Piao et al.'s scheme can only be used for a one-time group communication.

3.2 Replay Attack

If an attacker, Eve, intercepts the polynomial $F(x)$ sent by the group controller, she can easily mount the replay attack by replaying it. This is because group members do not verify the freshness of the group key. Assume that Eve has recorded the polynomial $F_{old}(x) = (x-SK_1)(x-SK_2)\dots(x-SK_n)+GK_{k,old}$. Following, we consider different scenarios associated with this attack.

Case 1. Assume that the group membership does not change after the group key $GK_{k,old}$ being compromised by Eve.

Assume that the group controller sends a new key distribution message $F_{new}(x) = (x-SK_1)(x-SK_2)\dots(x-SK_n)+GK_{k,new}$ to members of the same group, where $GK_{k,new}$ is the new intra-group key. The attacker, Eve, can replay the polynomial $F_{old}(x)$ corresponding to the group key $GK_{k,old}$

which has been compromised by Eve already. Obviously, all members in the group can retrieve the intra-group key GK_{k_old} and use the key GK_{k_old} to communicate with each other. However, Eve knows the content of all future communications.

Case 2. Assume that a new member has joined in the group after GK_{k_old} being used for the group communication; but the group key GK_{k_old} has not been compromised by Eve.

Assume that the group controller sends a new key distribution message $F_{new}(x) = (x-SK_1)(x-SK_2)...(x-SK_n)(x-SK_w)+GK_{k_new}$ when a new member, William, has joined in the group, where SK_w is William's secret key shared with the controller and GK_{k_new} is the new intra-group key. The attacker, Eve, can replay the polynomial $F_{old}(x)$ to all members in the group. After receiving the replayed message, only William cannot retrieve the corresponding intra-group key GK_{k_old} from $F_{old}(x)$. This is because that the replayed polynomial $F_{old}(x) = (x-SK_1)(x-SK_2)...(x-SK_n)+GK_{k_old}$ is generated before William joining in the group. Obviously, when William computes $F_{old}(SK_w)$ using his own secret key SK_w , he cannot get the same group key as other members' obtained. This will end up an unsuccessful group communication.

3.3 No Forward/Backward Secrecy

Assume that a dishonest user, David, who used to be a group member knowing the group key GK_{k_old} , attempts to obtain the content of communications which he is not authorized to.

3.3.1 No Forward Secrecy

Assume that David has stored the polynomial $F_{old}(x) = (x-SK_1)(x-SK_2)...(x-SK_n)+GK_{k_old}$ and known the key, GK_{k_old} , since he was a member of this group. Afterward, he just replays the polynomial disguised as a new key distribution message to all members in the group. He can easily learn the traffic of the group communications which he is not authorized to. This is because they will use the same key GK_{k_old} to communicate with each other. It is noteworthy that such attack is based on the assumption that the group membership does not change after David leaving the group. As a result, Piao et al.'s intra-group key distribution scheme [8] does not possess perfect forward secrecy.

3.3.2 No Backward Secrecy

Assume that David intends to deduce the previous intra-group key which he is not authorized to. He can launch the attack through following procedures.

Step 1. David intercepts the polynomial $F(x)$ sent by the group controller and then joins in the group. Assume that the polynomial he intercepted is $F(x) = (x-SK_1)(x-SK_2)...(x-SK_n)+GK_k$.

Step 2. After David joining in the group, the group controller gives a share key SK_D to him through a secure channel. In addition, the controller

generates a new intra-group key GK_{k_new} and constructs a new polynomial as $F_{new}(x) = (x-SK_1)(x-SK_2)...(x-SK_n)(x-SK_D)+GK_{k_new}$. The controller sends $F_{new}(x)$ to all members including David.

Step 3. Upon receiving the polynomial, David can use his own secret key SK_D to retrieve the intra-group key GK_{k_new} by computing $F_{new}(SK_D)$. Then, he computes a new polynomial $H(x) = (F_{new}(x) - GK_{k_new})/(x-SK_D)$.

Step 4. David can deduce the previous intra-group key GK_k by computing $F(x) - H(x)$. Obviously, the key he deduced is valid since $H(x) = (F_{new}(x) - GK_{k_new})/(x-SK_D) = (x-SK_1)(x-SK_2)...(x-SK_n)$.

Thus, Piao et al.'s intra-group key distribution scheme [8] does not possess perfect backward secrecy.

4 A Simple Modification

In this section, we provide a simple modified scheme to overcome security weaknesses as mentioned in last section. In order to prevent the internal attack as we have described previously, we use a composite number as the modulus i.e., $N = pq$, where p and q are large primes used in RSA scheme [9]. In addition, we add random challenges of group members to overcome the replay attack.

Similarly, we assume that in a group G_k with n members, all members have pre-shared their secret keys SK_i 's with the group controller. The group controller performs following procedures to distribute the intra-group key GK_k to all members.

Step 1. The group controller broadcasts a group communication message to all members.

Step 2. After receiving the message, each member randomly chooses a challenge C_t from ZN^* and sends it back to the controller.

Step 3. The group controller generates an intra-group key GK_k for all members in the group communication. Then, the controller uses all challenges and secret keys shared with the group members to generate a polynomial $F(x)$ with modulus N (i.e., $N = pq$, where p and q are large primes) as $F(x) = (x-(SK_1 \oplus C_1))(x-(SK_2 \oplus C_2))...(x-(SK_n \oplus C_n))+GK_k$.

Finally, the controller computes an authentication message as $Auth = h(GK_k)$ and sends it along with $F(x)$ to the members, where $h(\cdot)$ is a secure one-way hash function.

Step 4. Upon receiving the key distribution message, every member can use her/his own secret key SK_i and challenge C_i to retrieve the intra-group key GK_k by computing $F(SK_i \oplus C_i)$. After retrieving the intra-group key, every member also can authenticate the group key by checking $h(GK_k) = Auth$.

5 Discussions

Here, we analyze our modified scheme with respect to each security problem as we have mentioned in Section 3.

5.1 Security of Pre-Shared Secrets

In our proposed scheme, we replace the prime modulus p by the composite number N , where $N = pq$ as used in RSA [9]. Assume that a dishonest member, Ivy, in a group has received the polynomial $F(x) = (x-(SK_1 \oplus C_1))(x-(SK_2 \oplus C_2)) \dots (x-(SK_l \oplus C_l)) \dots (x-(SK_n \oplus C_n)) + GK_k$ from the group controller and retrieved the intra-group key GK_k . Ivy tries to obtain the secret keys SK_i 's of other group members shared with the controller by deducing another polynomial in Z_N^* as

$$H(x) = (F(x) - GK_k) / (x - (SK_l \oplus C_l)) \\ = (x - (SK_1 \oplus C_1))(x - (SK_2 \oplus C_2)) \dots (x - (SK_{l-1} \oplus C_{l-1}))(x - (SK_{l+1} \oplus C_{l+1})) \dots (x - (SK_n \oplus C_n)).$$

She needs to solve the roots of the polynomial $H(x) \equiv 0 \pmod{N}$ in order to find the secrets. In other words, Ivy needs to solve two separate equations as $H(x) \equiv 0 \pmod{p}$ and $H(x) \equiv 0 \pmod{q}$. Nevertheless, this approach is impossible since it is computationally infeasible to factor N (i.e., the factorization assumption in RSA [9]). It is noteworthy that Coppersmith has shown that finding small roots of an univariate polynomial equation modulo an integer N of unknown factorization is easy [3]. However, the secret in our application is at least 100 bits so it is not a "small solution". Thus, the algorithm described in [3] cannot be used to solve the pre-shared secrets of members in our proposed scheme.

5.2 Replay Attack

Assume that there is an adversary, Eve, who has intercepted messages transmitted publicly. If Eve intercepts the polynomial $F(x)$ sent by the group controller and attempts to mount the replay attack by replaying it, no matter whether the group membership has changed or not, this attack cannot work properly. This is because the polynomial $F(x)$ involves each member's random challenge C_i which is refreshed for each key distribution. The group members can verify the freshness of the group key. Following, we give an example to analyze the replay attack in detail.

Assume that the group controller sends a new key distribution message $F_{new}(x) = (x - (SK_1 \oplus C_{1,new}))(x - (SK_2 \oplus C_{2,new})) \dots (x - (SK_n \oplus C_{n,new})) + GK_{k,new}$ with the authentication message $Auth_{new} = h(GK_{k,new})$ to the members of the same group. Note that $GK_{k,new}$ is the new intra-group key and $C_{i,new}$'s are new challenges of all members used in current session. If Eve intends to launch the replay attack, she must replay the polynomial

$$F_{old}(x) = (x - (SK_1 \oplus C_{1,old}))(x - (SK_2 \oplus C_{2,old})) \dots \\ (x - (SK_n \oplus C_{n,old})) + GK_{k,old}$$

and the corresponding authentication message $Auth_{old} = h(GK_{k,old})$ which has been stored by her already. After receiving the replayed key distribution message, every member uses her/his own secret key SK_i and current challenge $C_{i,new}$ to compute the intra-group key $GK'_k = F_{old}(SK_i \oplus C_{i,new})$. Obviously, the computed GK'_k is different from $GK_{k,old}$ which was concealed in $F_{old}(x)$. Hence, every member can verify that this key is incorrect by checking $h(GK'_k) \neq Auth_{old}$ and then asks the controller to resend another key distribution message. As a result, the random challenges and the authentication message used in our proposed scheme can overcome the replay attack.

5.3 Forward/Backward Secrecy

Assume that an adversary (i.e., a dishonest member or an external attacker), Eve, who has compromised the group key $GK_{k,old}$ in the polynomial $F_{old}(x)$, attempts to obtain the content of communications that she is not authorized to. We consider different scenarios associated with forward/backward secrecy.

Case 1. Assume that Eve intends to destroy forward secrecy by replaying old key distribution messages $F_{old}(x)$ and $Auth_{old}$.

Assume that the group controller sends a new key distribution message $F_{new}(x) = (x - (SK_1 \oplus C_{1,new}))(x - (SK_2 \oplus C_{2,new})) \dots (x - (SK_n \oplus C_{n,new})) + GK_{k,new}$ and the authentication message $Auth_{new} = h(GK_{k,new})$ to members of the same group, where $GK_{k,new}$ is the new intra-group key and $C_{i,new}$'s are new challenges of all members used in current session. In order to make group members to use old key $GK_{k,old}$ to communicate with each other, Eve replays the polynomial $F_{old}(x)$ and $Auth_{old}$ disguised as a new key distribution message to all members in the group. By following our proposed scheme, this attack cannot work properly. As mentioned in Subsection 5.2, the key retrieved by every member is different from $GK_{k,old}$. This is because each member's random challenge C_i associated with the polynomial $F(x)$ is refreshed for each key distribution. Hence, all members would not use the retrieved key for group communication. The forward secrecy is achieved.

Case 2. Assume that Eve intends to destroy forward/backward secrecy by deducing previous/following intra-group key from $GK_{k,old}$ in the polynomial $F_{old}(x)$.

In our proposed scheme, the intra-group keys GK_k 's used in different sessions are all independent. Obviously, it is impossible to reveal other keys from a compromised key $GK_{k,old}$. Furthermore, if Eve intends to deduce the previous/following intra-group key from the polynomial $F_{old}(x)$, she may deduce another polynomial in Z_N^* (i.e., $N = pq$ as used in RSA [9]) as $H(x) = F_{old}(x) - GK_{k,old} = (x - (SK_1 \oplus C_{1,old}))(x - (SK_2 \oplus C_{2,old})) \dots (x - (SK_n \oplus C_{n,old}))$. Then, she tries to obtain the secret key SK_i of each group member shared with the group controller from $H(x)$. As explained in Subsection 5.1, it is computationally infeasible for solving the roots of the polynomial $H(x)$. Hence, Eve cannot obtain

the secret SK_i of each group member. Obviously, Eve cannot deduce the corresponding intra-group key from the previous/following polynomial without knowing group members' secret keys SK_i 's. As a result, our proposed scheme can achieve perfect forward/backward secrecy.

6 Conclusions

In this paper, we have described the security problems of Piao et al.'s intra-group key distribution scheme [8]. In their proposed scheme, a prime is used as the modulus for the polynomial operation. This setting cannot prevent a group member to obtain other members' secret keys shared with the controller. In addition, Piao et al.'s scheme is suffered from the replay attack and cannot achieve the objectives of both perfect forward and backward secrecy. We also provided a simple modified scheme to overcome these security problems. Detailed security analysis is also included.

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